A Pendulum Tuned Mass Damper

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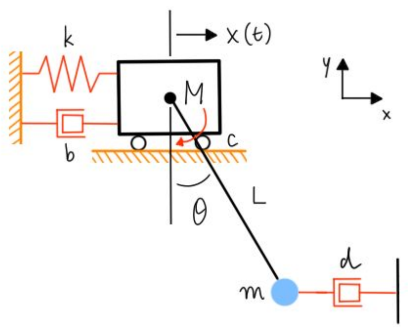
Introduction

Most of us hold the fundamental belief that buildings do not move. In contrast, the reality is that tall structures have a certain flexibility, so when designing a skyscraper, engineers have to take oscillations caused by strong winds and earthquakes into account.

How can we prevent or at least dampen these oscillations? In our project we are presenting and exploring one solution to the problem called a "Pendulum Tuned Mass Damper", or in short a "PTMD".

The Model

In our model we attempted to mimic the movements of a building with a PTMD.



We developed the equations for the model, and got these differential equations:

$$\ddot{x} = \frac{mg\sin\theta\cos\theta + mL\dot{\theta}^2\sin\theta + \dot{\theta}\cos\theta\left(\frac{C}{L} - dL\cos^2\theta\right) - kx - b\dot{x}}{m + M - m\cos^2\theta}$$

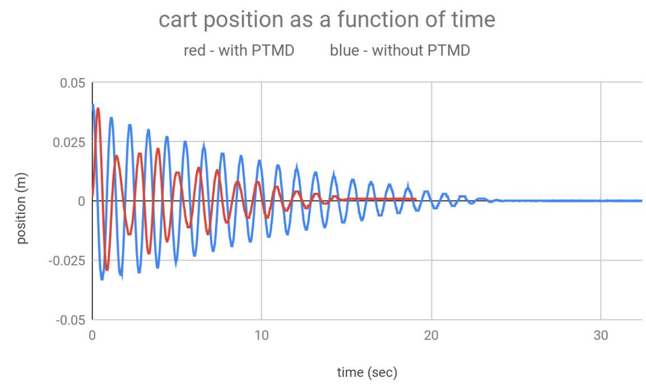
$$\ddot{\theta} = \frac{g(M+m)\sin\theta + \frac{\dot{\theta}(M+m)}{m}\left(\frac{C}{L} + dL\cos^2\theta\right) - K\cos\theta x - b\cos\theta\dot{x} + mL\dot{\theta}^2\sin\theta\cos\theta}{mL\cos^2\theta - L(M+m)}$$

In order to explore the equations, we used the Euler method.

Experiment & Results

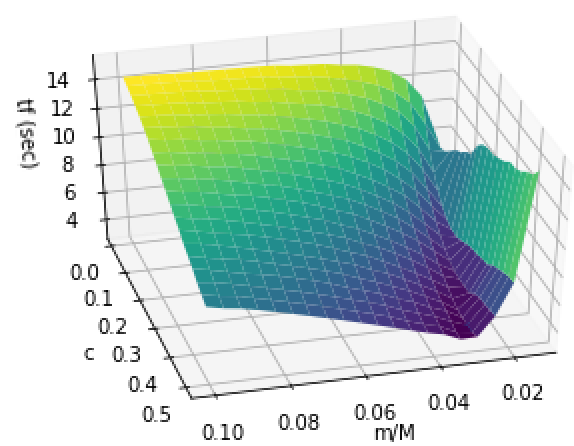
We conducted an experiment in order to confirm that our model works as expected.

A graph of the cart position as a function of time, with the PTMD and without:

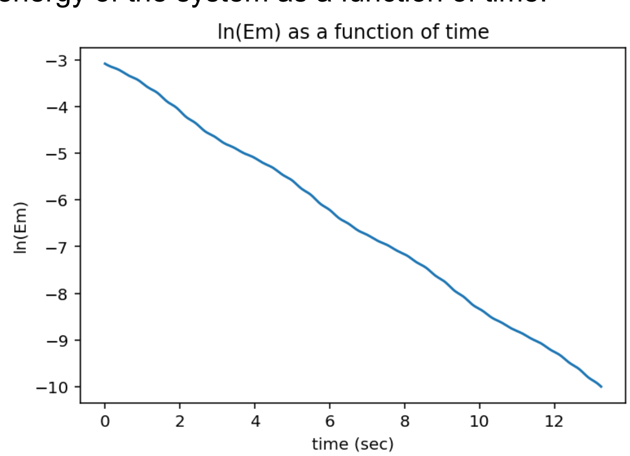


Additional Model Results

An example of a graph of the decaying time as a function of the rope damping and the pendulum/building mass ratio:



A graph of the natural logarithm of the mechanical energy of the system as a function of time:



As you can see, the energy decays exponentially.

Conclusions

Looking at our model and the results above, the main conclusion is that the PTMD is indeed effective when tuned. Moreover, we found that the system's total mechanical energy decays in an exponential manner.

In our search for the optimal values for the damping constants with which the dampening would be most effective, we found that there is in fact a set of values that maximizes the decay rate.

For Further Research

- Making the model more realistic -
    - Making the wind/earthquake a continuous force, and then tuning the pendulum to have a natural frequency identical to it
    - Adding another damping factor (d, for example)
    - Making it 3D
- Comparing the model to real buildings